

# Wall Temperature Estimation for Heated Underwater Bodies

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This study makes use of the results of similarity solutions of the laminar boundary layer equations with heat to formalize a design procedure for estimating wall temperature and heat flux values necessary to stabilize the boundary layer on a class of underwater bodies. In the use of the axisymmetric flow equations, a relation between the equivalent of the Hartree  $\beta$  for two-dimensional flow and pressure and radius gradient is used. The results of a trial computation are presented to show the validity of the procedure. Information is also presented for the estimation of laminar separation and transition locations on an axisymmetric body with a given wall temperature distribution.

## Nomenclature

$C_p$	= specific heat at constant pressure = $C_p^*/C_{p\infty}^*$
$E$	= Eckert number = $U_\infty^2/C_{p\infty}^* T_\infty^*$
$f$	= transformed stream function = $\psi(x,y)/(2\xi/R_e)^{1/2}$
$g$	= temperature ratio (= $T$ ) = $T^*/T_\infty^*$
$H$	= shape factor = $\delta^*/\theta$
$K$	= (thermal conductivity) = $k^*/k_\infty^*$
$M$	= pressure gradient parameter = $(x/U_e)(dU_e/dx)$
$P_\infty$	= freestream Prandtl number = $C_{p\infty}^* \mu_\infty^*/k_\infty^*$
$q^*$	= heat flux per unit area
$q_c$	= heat flux coefficient
$r$	= radius coordinate = $r^*/L^*$
$r_0$	= body radius = $r_0^*/L^*$
$Re$	= Reynolds number = $U_\infty^* L^* \rho_\infty^*/\mu_\infty^*$
$Re_x$	= Reynolds number = $U_\infty^* x^* \rho_\infty^*/\mu_\infty^*$
$Re_\delta^*$	= Reynolds number = $U_e^* \delta^* \rho_\infty^*/\mu_\infty^*$
$T$	= temperature = $T^*/T_\infty^*$
$U_e$	= velocity at edge of boundary layer = inviscid velocity at body surface = $U_e^*/U_\infty^*$
$u$	= velocity in $x$ direction = $u^*/U_\infty^*$
$v$	= velocity in $y$ direction = $v^*/U_\infty^*$
$x$	= arc length distance along body = $x^*/L^*$
$y$	= distance normal to body = $y^*/L^*$
$\beta$	= pressure gradient parameter = $(2\xi/U_e)(dU_e/d\xi)$
$\delta$	= boundary layer thickness (dimensional)
$\delta^*$	= displacement thickness (dimensional)
$\eta$	= transformed $y$ coordinate = $(R_e/2\xi)^{1/2} U_e r_0 dy$
$\theta$	= momentum thickness (dimensional)
$\Lambda$	= Pohlhausen parameter = $(\delta/\nu_\infty^*)(dU_e^*/dx^*)$
$\lambda$	= radius gradient parameter = $(2x/r_0)(dr_0/dx)$
$\mu$	= dynamic viscosity = $\mu^*/\mu_\infty^*$
$\nu$	= kinematic viscosity = $\nu^*/\mu_\infty^*$
$\xi$	= transformed $x$ coordinate ( $d\xi = U_e r_0^2 dx$ )
$\rho$	= mass density = $\rho^*/\rho_\infty^*$
$\tau^*$	= shear stress
$\tau_c$	= shear stress coefficient
$\psi$	= stream function = $\psi^*/\rho_\infty^* L^{*2} U_\infty^*$

## Subscripts

crit	= critical value
$e$	= evaluated at the edge of the boundary layer
trans	= transition value
$w$	= evaluated at the wall
$\infty$	= evaluated in the freestream

## Superscripts

( ) <sup>*</sup>	= dimensional quantity
( )'	= differentiation with respect to $\eta$

## Dimensional Reference Quantities

$L^*$	= reference length
$T_\infty^*$	= freestream temperature
$U_\infty^*$	= freestream velocity
$\mu_\infty^*$	= freestream viscosity
$\rho_\infty^*$	= freestream density

## Introduction

THE reporting of this study is meant to fulfill a need for providing calculated data and techniques for a designer wishing to determine the surface heat requirements for stabilizing the boundary layer of an axisymmetric body moving in water. Because of the infinite possible variations in body profiles, it becomes necessary to resort to similar solutions of the boundary layer equations and to restrict the choice of bodies that should be considered. The type of body to which heat can be effectively added is one which has gradually varying surface pressure and body radius over a major portion of the body length. By choosing a range of values of a few parameters and proceeding with the boundary layer calculations, enough information becomes available to effect a design procedure.

The procedure which evolved is based on the work originally reported in Ref. 1 and then greatly simplified in Ref. 2. The present work parallels the studies reported in Refs. 3 and 4. In Ref. 3, a relationship between the transition Reynolds number and the shape factor is developed on the basis of stability analyses of wedge flows and heat transfer in water and the  $e^9$  type of stability criterion. In Ref. 4, Thwaites' integral method is extended to the case of heated laminar boundary layers in water and used to calculate the laminar boundary layer properties including heat transfer.

## Development of Equations and Their Solutions

The conservation equation for mass, momentum, and thermal energy in a steady boundary layer can be written in dimensionless form as

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0$$

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \rho U_e \frac{dU_e}{dx} + \frac{1}{R_e} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right)$$

and

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = -E \mu U_e \frac{dU_e}{dx} + \frac{1}{P_R R_e} \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right)$$

(1)

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Equations (1) do not include transverse curvature terms; i.e., derivatives of  $r (=r_0(x) + y \cos \phi)$  with respect to  $y$  are considered small. In effect, then,  $r = r_0(x)$ . Also, the Eckert number,  $E$ , when evaluated for water at expected conditions in the free-stream is a small quantity and the term in which it appears can be dropped.

If Eqs. (1) are transformed according to the Mangler-Levy-Lees transformation,

$$d\xi = U_e r_0^2 dx \quad (2)$$

$$d\eta = (R_e/2\xi)^{1/2} U_e r_0 dy \quad (3)$$

and a reduced stream function,

$$\psi(x, y) = (2\xi/R_e)^{1/2} f(\xi, \eta) \quad (4)$$

and the parameter

$$\beta = \frac{2\xi}{U_e} \frac{dU_e}{d\xi} \quad (5)$$

are introduced, Eqs. (1) reduce to

$$\begin{aligned} (\rho \mu f''')' + f f'' + \beta(1/\rho - f'^2) &= 2\xi(f' f_\xi' - f'' f_\xi) \\ (1/P_R)(\rho k g')' + C_p f g' &= 2\xi C_p(f' g_\xi' - g' f_\xi) \end{aligned} \quad (6)$$

In Eqs. (6),  $g$  is used in place of  $T$  to correspond with the nomenclature of other presentations. If the functions  $f$  and  $g$  are assumed to be independent of the transformed coordinate

$\xi$ , the following similar-type homogeneous equations result:

$$\begin{aligned} (\rho \mu f'')' + \beta(1/\rho - f'^2) + f f'' &= 0 \\ P_R C_p f g' + (\rho k g)' &= 0 \end{aligned} \quad (7)$$

The attendant boundary conditions are

At  $\eta = 0$ :

$$f = f' = 0 \quad g = T_w$$

At  $\eta \rightarrow \infty$ :

$$f' \rightarrow 1 \quad g \rightarrow I$$

As developed in Ref. 2, the parameter  $\beta$  of Eqs. (7) is related to two parameters  $M = (x/U_e)(dU_e/dx)$  and  $\lambda = (2x/r_0)(dr_0/dx)$  by the equation

$$\beta = \frac{2M}{M + \lambda + 1} \quad (8)$$

thus reducing by one the parameters relating the solution of Eqs. (7) to the physical picture introduced by the body and the potential flow over that body.  $\lambda$  values are known for specific locations on the body from the details of the contour and  $M$  can be determined from either a measured pressure distribution or computations from the Douglas-Neumann procedure.

A finite-difference procedure due to H. B. Keller<sup>5</sup> was used in solving Eqs. (7). The procedure, known as the box method, was developed by Keller and applied successfully by Cebeci and Smith<sup>6</sup> to a wide variety of problems. The physical properties of water,  $C_p$ ,  $\rho$ ,  $\mu$ , and  $k$  are a function of temperature and are known from information presented in Ref. 7.

### Results of Boundary Layer Computations

The boundary layer computations were performed for a parametric study in which the parameters were  $\beta$  and  $\Delta T (=T_w - T_\infty)$ . In all cases, the freestream temperature,  $T_\infty$ , was kept at 60°F. A tabular summary of the results of this parametric study is found in Ref. 2. Plots of these data are displayed here in Figs. 1-5. The data are similar to those presented in Refs. 3 and 4, except that in those cases a slightly different  $T_\infty$  (=67°F) was used and that a greater negative range of  $\beta$  is used here.

The boundary layer thicknesses,  $\delta^*$  and  $\theta$ , are defined in the sense of two-dimensional definitions,

$$\delta^* = \int_0^\infty (1 - \rho u) dy^* \quad (9)$$

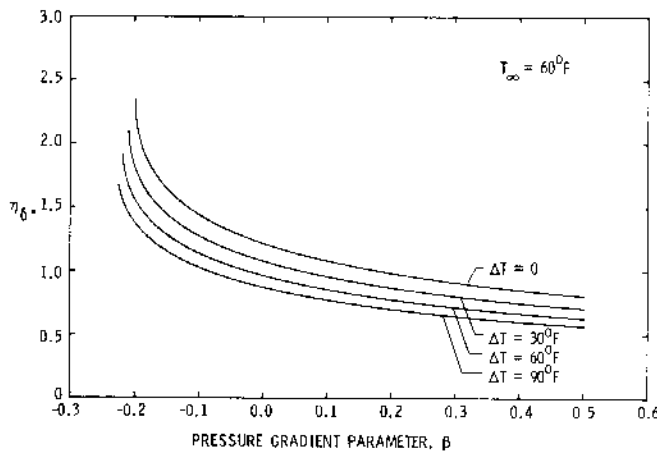


Fig. 1 Variation of  $\eta\delta^*$  with  $\beta$  parameter.

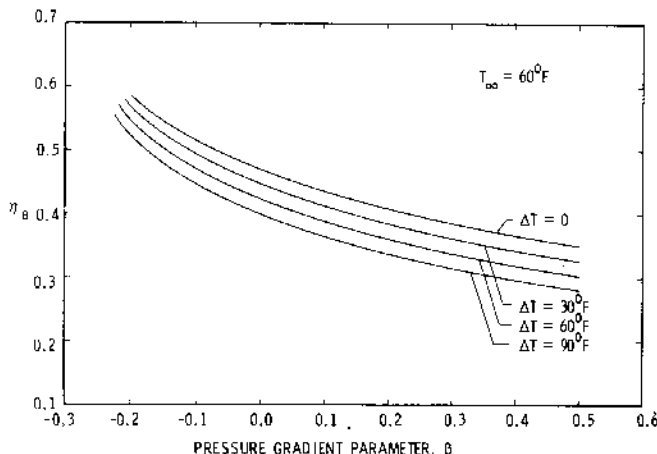


Fig. 2 Variation of  $\eta\theta$  with  $\beta$  parameter.

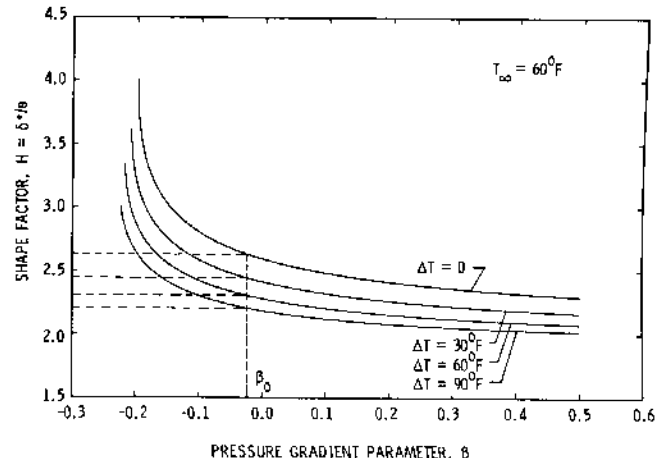
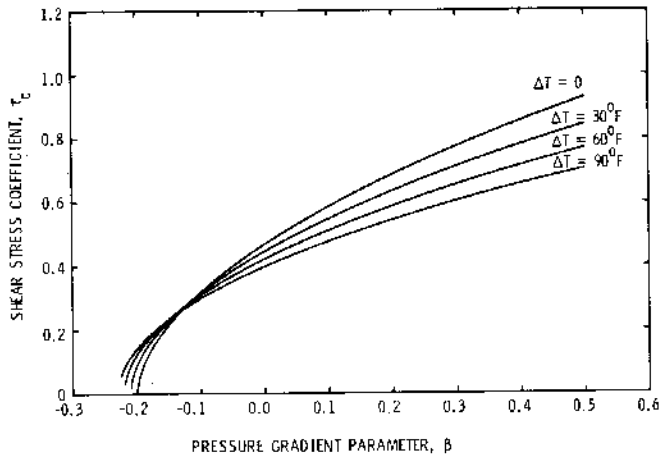
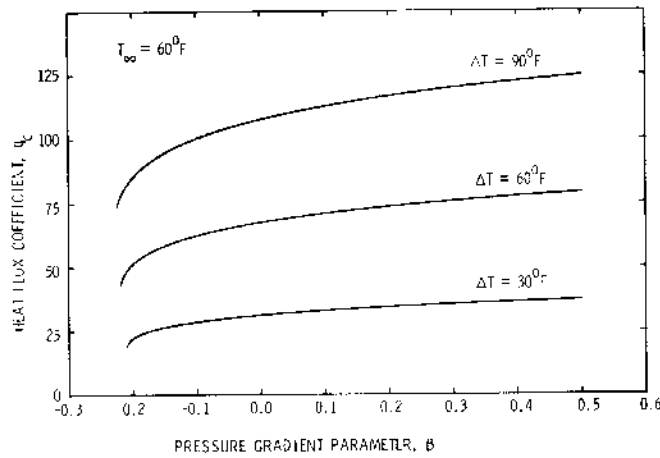


Fig. 3 Variation of shape factor with  $\beta$  parameter.

Fig. 4 Variation of  $\tau_c$  with  $\beta$  parameter.Fig. 5 Variation of  $q_c$  with  $\beta$  parameter.

and

$$\theta = \int_0^\infty \rho u (1-u) dy^* \quad (10)$$

This leads eventually to slight differences when comparing with thicknesses resulting from axisymmetric considerations. Little difference in the shape factor,  $H (= \delta^*/\theta)$ , should result, however, when the location on the body under consideration has a small boundary layer thickness compared with the body radius. For considerations of a laminar boundary layer this is, of course, generally true.

The quantities  $\eta_\delta^*$  and  $\eta_\theta$  shown in Figs. 1 and 2 must be transformed to get  $\delta^*$  and  $\theta$  as follows:

$$\delta^* = (\beta/M)^{1/2} (\nu_\infty^* x^*/U_\infty^*)^{1/2} \eta_\delta^* \quad (11)$$

$$\theta = (\beta/M)^{1/2} (\nu_\infty^* x^*/U_\infty^*)^{1/2} \eta_\theta \quad (12)$$

The shear stress and heat flux quantities,  $\tau_c$  and  $q_c$ , must likewise be transformed as follows to get  $\tau^*$  and  $q^*$ :

$$\tau^* = \mu_\infty^* U_\infty^* (U_\infty^*/\nu_\infty^* x^*)^{1/2} (M/\beta)^{1/2} \tau_c \quad (13)$$

$$q^* = k_\infty^* (U_\infty^*/\nu_\infty^* x^*)^{1/2} (M/\beta)^{1/2} q_c \quad (14)$$

### Critical Reynolds Number Correlation

Stability information in terms of the critical Reynolds number,  $R_{\delta^* \text{crit}}$ , is available for two-dimensional flow both for the case of the unheated boundary with pressure gradient and for the flow over a flat plate with heating at the wall. The results of the unheated case are found in Ref. 8 as critical Reynolds

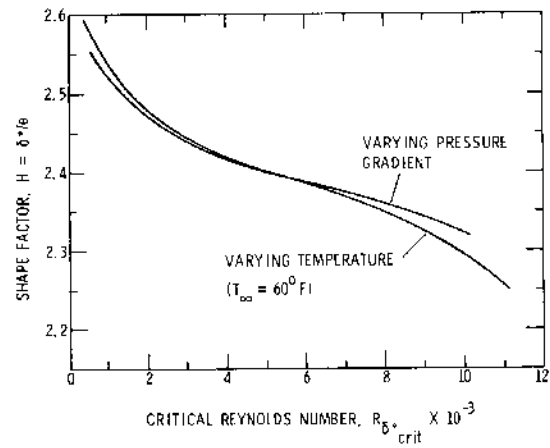


Fig. 6 Correlation of shape factor with critical Reynolds number.

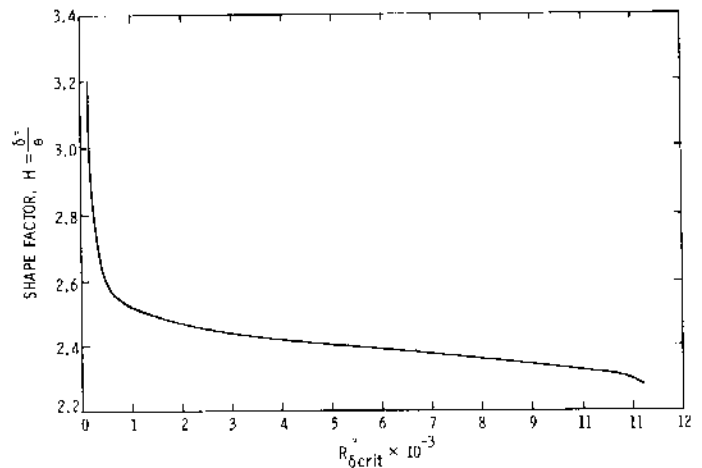


Fig. 7 Shape factor vs critical Reynolds number.

number vs the Pohlhausen parameter,  $\Lambda = (\delta/\nu^*) (dU_\infty^*/dx^*)$ , and the heated case results were obtained from Ref. 7 as critical Reynolds number vs wall temperature for a fixed  $T_\infty^* = 60^\circ F$ . The  $\Lambda$  and  $\Delta T$  parameters were translated into the shape factor  $H = \delta^*/\theta$ , so that in each case  $R_{\delta^* \text{crit}}$  could be plotted against  $H$ . The curves in Fig. 6 are the result. It can be seen that there is fair correlation, indicating that the stability of the laminar boundary layer is strongly dependent upon  $H$ , regardless of whether it is obtained by favorable pressure gradient or by heat. A similar correlation is reported in Ref. 9 and is also used to advantage in Ref. 3.

This correlation provides the basis for the development of the type of design information which will be presented here. Although based on two-dimensional stability information, the correlation is considered valid for the axisymmetric case because the stability equations, under the assumption that the boundary layer thickness is small compared to the local body radius, are the same for both cases. It must also be stated that the curve that results from  $\Delta T$  is based on a constant wall temperature (i.e., no variation in wall temperature with distance in the flow direction). As shown in Ref. 10, a varying  $\Delta T$  can alter the  $R_{\delta^* \text{crit}}$  curve from the form used here.

The curve in Fig. 6 for varying pressure gradient is repeated in Fig. 7 and extrapolated in the low  $R_{\delta^* \text{crit}}$  range. This curve is used subsequently to represent the variation of  $H$  with  $R_{\delta^*}$  regardless of how  $H$  is obtained.

### Determination of Body Temperature Distribution

The procedure outlined here follows the approaches presented in Refs. 1 and 2. Two criteria were chosen to insure the maintenance of laminar flow: 1) the provision of just enough heat to keep the Reynolds number (based on displacement

Table 1 Comparison of laminar separation results for two-dimensional unheated case

$f_w''$ (ARL)	$f_w''$ (Keller-Cebeci)	$\beta$ (ARL)	$\beta$ (Keller-Cebeci)	$\beta$ (Smith)
+ .00064472 - .00076458 } .0551894 .0857052 .128638 .190780 .239736 .319270 .400322 .469600	0 .05517 .08570 .12864 .19078 .23974 .31927 .40032 .46960	-.19891 -.195 -.190 -.180 -.160 -.140 -.100 -.050 0	-.20259 -.19528 -.19023 -.18025 -.16016 -.14024 -.10017 -.05031 -.00031	-.198838 -.195 -.190 -.180 -.160 -.140 -.100 -.050 0

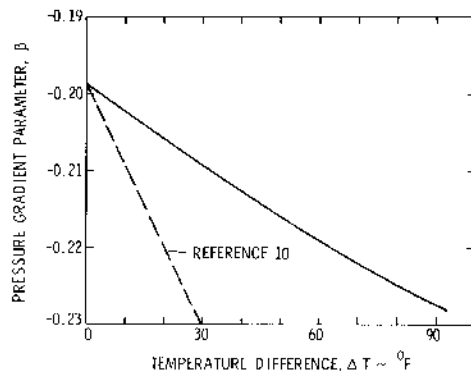


Fig. 8 Laminar separation limit curve.

thickness) equal to the critical Reynolds number and; 2) the provision of enough heat so that the peak critical Reynolds number is maintained. These are referred to as "minimum-heat" and "maximum-heat" conditions, respectively. The minimum-heat criterion implies that, for a particular free-stream velocity, enough heat is added to make the operating Reynolds number  $R_{\delta^*}$ , equal to the critical value. This should insure that there will never be any amplification of waves in the laminar boundary layer. The maximum-heat criterion fixes the  $R_{\delta^*}$  at its maximum value. Whether there is amplification depends upon the freestream velocity being high enough to have the operating  $R_{\delta^*}$  exceed the maximum  $R_{\delta^*crit}$ .

In implementing the temperature hunting procedure, the calculated data used for Figs. 1-5 can be filed and then recovered by the computer for interpolation purposes. The geometry and potential flow pressure distributions for a body will be known so that an  $M$  and a  $\lambda$ , and consequently a  $\beta$ , can be determined for each point on the body under consideration. This will be designated as  $\beta_0$  for a particular body point. The  $\Delta T^*$  required at a body station can be determined by applying one of the criteria mentioned and interpolating the data to get appropriate quantities corresponding to  $\beta_0$ . In the minimum heat case, the procedure would be as follows:

1) Pick off  $H_0$  vs  $\Delta T^*$  values corresponding to  $\beta_0$  as illustrated by the procedure pictured in Fig. 3. These values can be represented by curve fits and intermediate values are extracted from these fits. All the other quantities ( $\eta_{\delta^*}$ ,  $q_c$ ,  $\tau_c$ ) can be similarly handled so that a fitted curve for each of these is known as a function of  $\Delta T^*$  for a particular  $\beta_0$ .

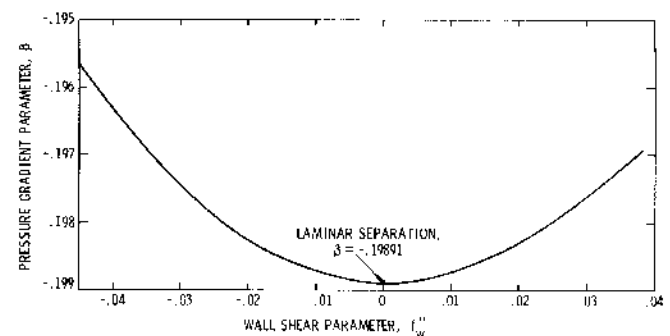
2) First choose  $\Delta T^* = 0$ . The  $\eta_{\delta^*}$  corresponding then to  $\Delta T^* = 0$  and  $\beta_0$  can be used to determine  $\delta^*$  for a particular freestream velocity using Eq. (11).

3) Calculate  $R_{\delta^*}$  by means of

$$R_{\delta^*} = \delta^* U_{\infty}'' / \nu_{\infty} \quad (15)$$

This is the operating  $R_{\delta^*}$ .

4) Enter Fig. 7 with  $R_{\delta^*}$  from step 3 to determine a required  $H$ . By entering with the operating  $R_{\delta^*}$ , we are saying that, in

Fig. 9 Variation of  $\beta$  parameter with wall shear parameter for Falkner-Shan flows.

order for this to be the  $R_{\delta^*crit}$ , we must produce a corresponding value of  $H$ .

5) The  $H$  required from step 4 can be used with  $H_0$  vs  $\Delta T^*$  curve of step 1 to determine the required  $\Delta T^*$ . This required  $\Delta T^*$  now will change the value of  $\eta_{\delta^*}$ , originally determined in step 2 for  $\Delta T^* = 0$ .

6) Repeat steps 2-5 until the  $\Delta T^*$  required converges within a desired accuracy.

7) Enter the  $q_c$  and  $\tau_c$  vs  $\Delta T^*$  curves established for  $\beta_0$  and get the  $q$  and  $\tau$  values from Eqs. (13) and (14).

For the maximum heat case, a slightly different procedure is as follows:

1) Assume  $H = 2.29$ . This corresponds approximately to the point where  $R_{\delta^*crit}$  reaches its maximum as evidenced in Ref. 7. This is not a very precise number and it could be as low as  $H = 2.2$ . Adding heat beyond the value that produces the maximum  $R_{\delta^*crit}$  will result in  $R_{\delta^*crit}$  becoming smaller and thus be counterproductive.

2) Enter the curve  $H_0$  vs  $\Delta T^*$ , determined in step 1 of the minimum-heat procedure, and extract the  $\Delta T^*$  required for  $H_0 = 2.29$ .

3) Extract  $q_c$  and  $\tau_c$  for  $\Delta T^*$  of step 2 from curves of  $q_c$  and  $\tau_c$  vs  $\Delta T^*$  also determined in step 1 of the minimum-heat procedure and again convert to  $q$  and  $\tau$  via Eqs. (13) and (14).

### Laminar Separation

An attempt was made to obtain limits of  $\beta$  for different  $\Delta T^*$  values beyond which laminar separation would occur. Using the criterion that the skin friction vanishes at the point of separation, curves of  $\tau_c$  vs  $\beta$  were extrapolated to obtain the desired limits. The computer program will not calculate a solution to the boundary layer equations at  $\tau = 0$ , so that extrapolation is necessary. Figure 8 is the result of those extrapolations. For a given local temperature difference, laminar separation will occur for values of  $\beta$  below the curve. The relatively small effect of temperature difference on laminar separation is apparent. A curve such as this was given in Ref. 11 and is plotted in part in Fig. 8.

As a check on the accuracy of the calculated separation  $\beta$ , an attempt was made to get as close to separation as possible

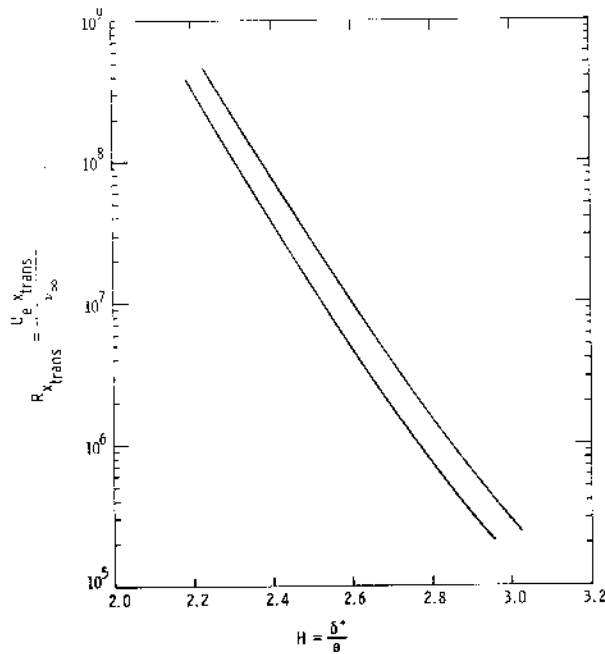


Fig. 10 Transition Reynolds number variation with shape factor from  $e^9$  stability calculations.

Table 2 Transition Reynolds number variation with shape factor

$H$	$R_{x_{trans}}$	$\log_{10}(R_{x_{trans}})$
2.20000	0.32909E 09	8.51732
2.25000	0.19074E 09	8.28045
2.30000	0.11067E 09	8.04405
2.35000	0.64390E 08	7.80882
2.40000	0.37625E 08	7.57548
2.45000	0.22117E 08	7.34473
2.50000	0.13100E 08	7.11730
2.55000	0.78324E 07	6.89390
2.60000	0.47341E 07	6.67524
2.65000	0.28976E 07	6.46204
2.70000	0.17989E 07	6.25501
2.75000	0.11346E 07	6.05486
2.80000	0.72829E 06	5.86231
2.84999	0.47651E 06	5.67808
2.90000	0.31832E 06	5.50287
2.95000	0.21747E 06	5.33741

for the unheated two-dimensional case. This is the solution for Falkner-Skan flows where  $\beta$  is now the Falkner-Skan  $\beta$ . These results were then compared with those presented in Ref. 6 and the comparison is shown in Table 1. The  $f_w''$  values, representing the slope of the velocity profile at the wall, and the corresponding  $\beta$  values are compared. Richardson's extrapolation<sup>6</sup> was used to get the ARL values of  $f_w''$ .

As can be seen, the Smith and ARL values compare favorably near laminar separation. The Keller-Cebeci values of  $\beta$  at laminar separation does not appear to plot smoothly with the rest of the values listed and thus appears to be in error. A plot of  $f_w''$  vs  $\beta$  for the ARL results appear in Fig. 9. Values corresponding to reverse flow also appear in this plot.

### Transition

The prediction of transition for a given temperature distribution on a body follows the same lines as presented in Ref. 1. The basis for the method used here is the plot of a band of calculated data supplied through the courtesy of A. M. O. Smith (see Fig. 10). The band marks the range of values of  $R_{x_{trans}}$  vs  $H$  that were obtained from  $e^9$  stability calculations performed for a variety of heated and unheated wedges. This kind of data also appears in Ref. 3. It would be logical to

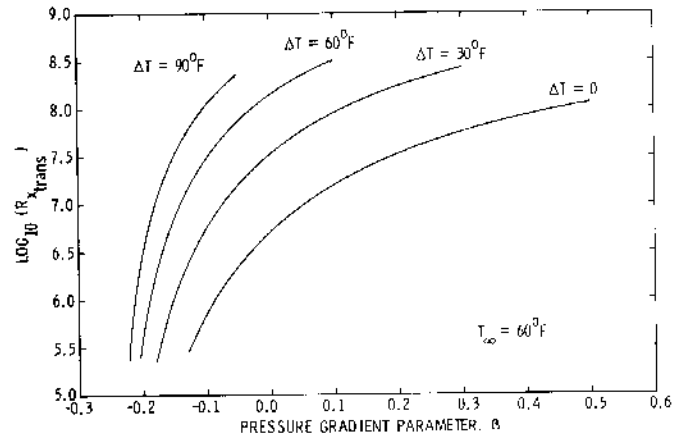


Fig. 11 Calculated transition Reynolds number data.

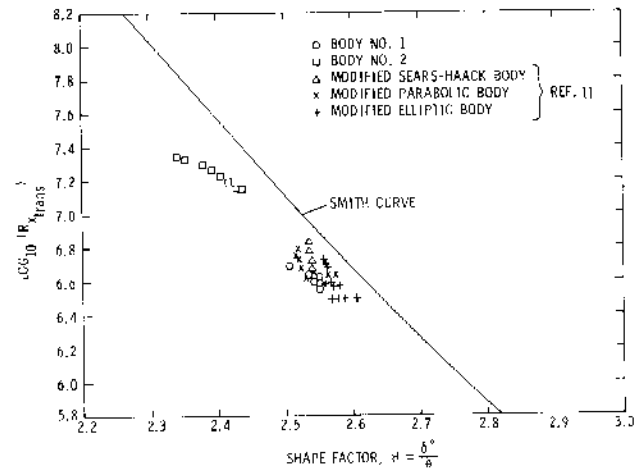


Fig. 12 Transition Reynolds number variation with shape factor.

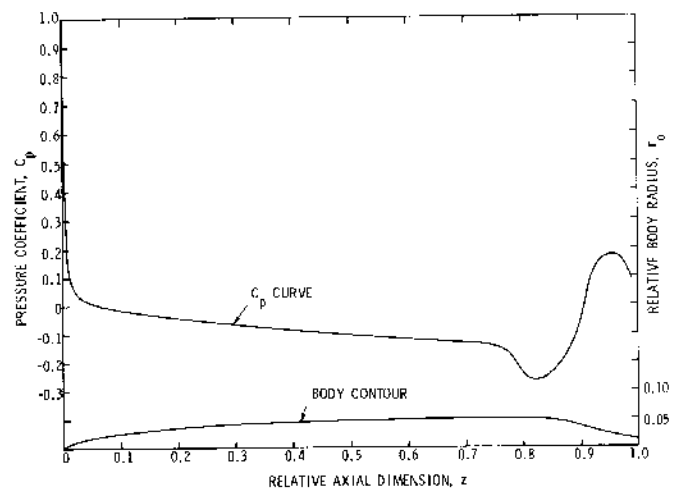


Fig. 13 Body contour and pressure distribution for trial computation.

choose the lower bound of this band as the transition criterion. Values for such a curve are given in tabular form in Table 2. Translating  $H$  into  $\beta$  and  $\Delta T^*$  by way of similarity solutions, the plots shown in Fig. 11 result. These curves indicate the transition Reynolds number that would correspond to particular values of  $\beta$  and  $\Delta T^*$ .

As stated in Ref. 1, the curves used as the criterion for transition is generally optimistic; that is, it predicts transition at a higher Reynolds number than some available unheated body transition data would indicate. This is evidenced by the experimental points plotted in comparison with the transition

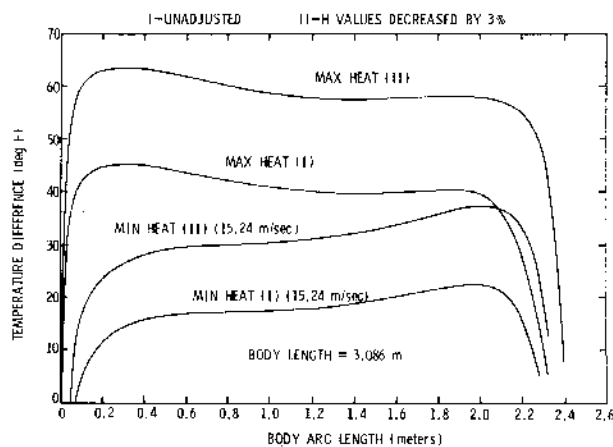


Fig. 14 Temperature distribution from trial computation.

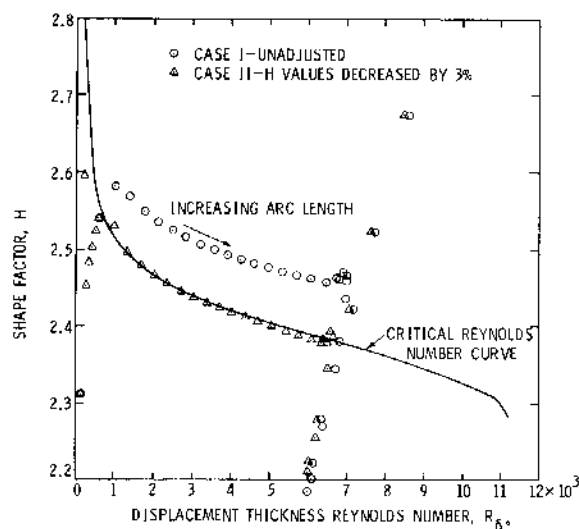


Fig. 15 Reynolds number comparison for minimum temperature distribution.

curve in Fig. 12. The data points are taken from Ref. 12 and the  $H$  values for these data were obtained from information in Fig. 3.

### Trial Computation

In order to check the validity of using the scheme developed previously in this paper, a trial computation was performed. The body used for this computation was the one employed in the horizontal buoyancy study reported in Ref. 13. Plots of the body contour and  $C_p$  distribution appear in Fig. 13. This body, with its moderately favorable pressure gradient over a larger portion of its length, is typical of the kind of underwater body whose transition might be effectively delayed by use of surface heating.

The kinds of temperature difference distributions that were estimated are shown in Fig. 14. Note that there are two sets of maximum and minimum distributions. The set labeled "I" resulted from applying the scheme without any adjustments.

Figure 12 suggests that, if the Smith curve were moved to the left by decreasing the  $H$  values of the curve by 3%, most of the experimental data would fall just on or above the curve. The  $H$  values for the experimental points were obtained from the solution of Eq. (7), whereas the computations on which the original band of computed data was based came from the solution of the nonsimilar equations. It has been observed that, at least for bodies of the type appearing in Fig. 13, the shape factors calculated via the nonsimilar equations are generally about 3% higher than those obtained from the

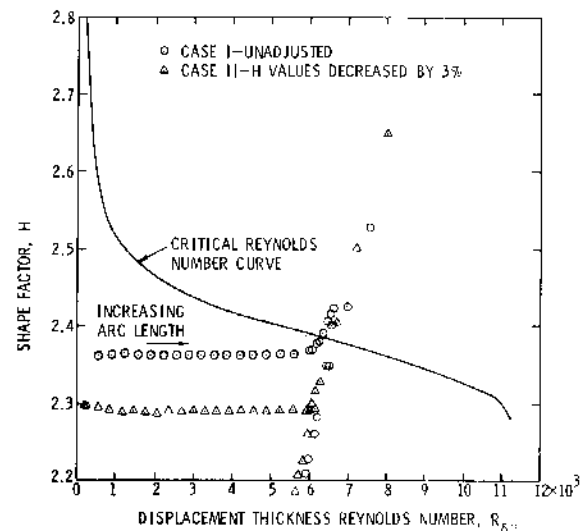


Fig. 16 Reynolds number comparison for maximum temperature distribution.

similar solutions. If one now moves the curve of Fig. 12 to the left by reducing the  $H$  values by 3% and uses this as the transition criterion, one would expect that the  $H$ 's calculated by way of similar solutions would give transition Reynolds number values close to values that are consistent with experimental and  $e^9$  calculated results.

Surprisingly, a 3% change in  $H$  causes a considerable change in temperature difference, as indicated by the curves labeled "II" in Fig. 14. This change is further reflected in the  $R_{\delta^*}$  calculated by the nonsimilar boundary layer computations in the Transition Analysis Program System (TAPS) described in Refs. 14 and 15. Points showing the adjusted and unadjusted values of  $R_{\delta^*}$  along the body for a freestream velocity of 15.24 m/s are seen in Fig. 15 for the minimum-heat case. These points are superimposed with the  $H$  vs  $R_{\delta^*}$  curve from Fig. 7. It can be seen that, for case II, the points follow the critical curve very closely in the region where heat is added. Finally, similar adjusted and unadjusted cases for the maximum heat addition are shown in Fig. 16, again for a freestream velocity of 15.24 m/s. As one would expect, both sets of points are below the critical curve until the region on the body where abrupt changes in  $C_p$  occur is reached.

As one would also expect, when the TAPS stability analysis was applied to the two cases for maximum heat for a freestream velocity of 15.24 m/s, the  $e^9$  amplification was not encountered until the region of strong adverse pressure gradient was reached. For minimum heat the same was true for the adjusted case (II) whereas, for the unadjusted case (I), transition was indicated a short distance back on the nose.

### Conclusions

The technique presented in this paper permits one to estimate, in a relatively easy manner, the temperature distribution necessary to stabilize the flow over a particular class of axisymmetric body. This technique also provides a means for approximately determining the local heat flux, skin friction, the laminar separation point, and the point of transition for a given temperature distribution on an axisymmetric body.

All these estimates can be made by using the values of the various quantities presented in the tables in Ref. 2 or the curves provided here and by following the interpolative procedures and criteria that have been outlined. A simple computer program to do this can be easily written.

A check on the validity of the scheme was established by comparing some experimental data with a band of computed  $e^9$  information and by applying TAPS to a particular representative axisymmetric body.

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